Finite Element Modelling of the Effect of Cutting Speed on Machining NST 37-2 Steel

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Abstract

The Nigerian Steel (NST) 37-2 has vast applications as a substitute for foreign steels in Nigeria, but, its reliability, the effects of induced residual stress in the material, optimum cutting parameters, efficient process planning, chip formation process and surface roughness of the machined products are some of the challenges faced in the industry. In this study, the finite element modelling and simulation of orthogonal cutting of NST 37-2 steel was carried out, and the effect of cutting speed was investigated to address informed predictability on the material. The finite element simulations were conducted by using the ABAQUS software. The Johnson-Cook plastic and damage models were employed together with arbitrary Lagrangian-Eulerian (ALE) formulations. The results showed mainly continuous chip formation for all simulations. It was observed that the cutting and the thrust forces decreased with increase in cutting speed. Also, the temperature of the shear zone region increased with increase in cutting speed.

Keywords: Finite Element Modelling (FEM), machining, Cutting speed, NST 37-2 Steel

1. INTRODUCTION

The Nigerian steel (NST) 37-2 is one of the commercial carbon steel grades produced by Nigeria steel industry, which represents about 75 percent volume of production. This grade of steel is produced from diverse scrap metals and iron ore imported into the country from various uncoordinated sources like Malaysia, Brazil, China, etc. (Malomo et al., 2010). This steel is commonly used in building construction as structural members and machine components in production.

The Nigerian steel (NST) 37-2 has vast applications as a substitute to foreign steels in Nigeria. There is increasing need for efficient machining and better manufacturing of this steel at low cost. The effects of induced residual stress in the work piece, optimum cutting parameters, efficient process planning, chip formation process and surface roughness of the machined products etc. are some of the challenges faced in the industry.

The experimental approach to study machining process is expensive and time consuming, especially when a wide range of parameters are included, like tool geometry, materials, cutting conditions and so on. Because of these difficulties, alternative approaches have been developed, such as simulations where numerical methods are used. Among these numerical methods, the Finite Element Method (FEM) has proved to be useful for such purpose (Cenk, 2009). Thus there is a need to introduce this method to investigate the orthogonal machining of NST 37-2 steel with the hope to promote the rapid adoption of its applications in machining industry in Nigeria.

The FEM can be applied in metal machining to predict optimum cutting parameters such as cutting speeds, temperature, cutting forces, rake angles, deformations in work piece material and the cutting tools.

Most commonly used finite element software are ABAQUS™, ANSYS™ and DEFORM2D™. "They are important tools suitable to solve linear and nonlinear problems, dynamic or static problems, contact between solids and ability to model large changes in the form of solids" (Everton, 2014).

Kovac *et al.* (2011) presented a computer modelling of tool deformation and thermal load during turning operation using finite element method. They found out that deformation of cutting tools is greatly caused by cutting temperature and cutting forces.

Ali et al. (2013) simulated the orthogonal milling of Ti6Ai4V alloy and predicted the cutting forces of high speed milling by varying the feed rate and analyzed the effect on surface roughness and feed cutting force.

Residual stress and shear strain are induced into the work piece and these determine its performance and fatigue strength. Zhang et al. (2011) studied the limiting shear stress at the tool chip interface and analyzed the effect of the coefficient of friction, cutting speed and tool rake angle. A similar approach of finite element method was employed to study the effect of rake angle in the distribution of stress and strain energy of AISI 1045 steel (Everton, 2014). It was discovered that the area of Von Mises Stress in the primary deformation zone decreased as the tool rake angle increased.

Fadare and Asafa (2007) developed a predictive model to determine the optimum cutting parameters for

- There is constant friction at the tool- chip interface and at the tool-work piece interface.
- The machine tool is perfectly rigid and no influence of machine tool dynamics on machining is considered.
- The work piece is at reference temperature (20^0C) at the beginning of simulation.
- Cutting is performed in air (dry machining) and no coolants are used. Adiabatic temperature boundary conditions were assumed. (Dhananjay, 2004).

Tool Modelling

The tool material was selected as uncoated carbide tool (Tounsil et al., 2002). Thermal and mechanical properties of the tool with geometric variables of the tool are given in Table 2. Finite element mesh of the tool was modelled using 1120 nodes and 1057 elements. Iso-parametric quadrilateral elements with structured mesh smoothing algorithms were used for the analysis. This design is shown in Figure 1.

The mechanical and thermal boundary layers in the contact region and within the localized shear bands were also smoothed by these meshings. Heat exchange is defined on the boundaries D-A and A-B. Boundaries B-C and D-C are sufficiently away from cutting edge therefore their temperature is fixed at 20 ºC.

Table 2: Thermal and Mechanical Properties of Uncoated Carbide Tool.

Properties	Amount
Density	14900
Elastic Modulus, E (GPA)	650
Poisson's Ratio	0.25
Thermal Conductivity (w/mºC)	79
Specific heat (J/Kg^{0}C)	138
Rake Angle, α (^{0})	0
Clearance Angle, c (^{0})	5
Tip Radius, rt (mm)	0.05

Source: (Tounsi *et al.*, 2002)

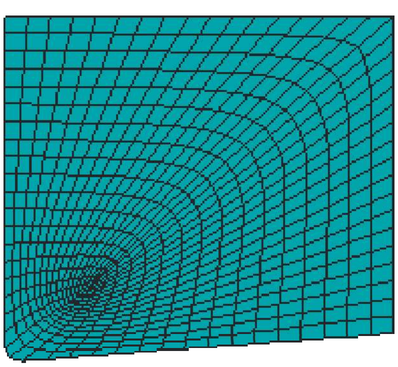

Figure 1: Mesh design of the tool

Work piece Modelling

The work material was considered as deformable. Accurate and reliable flow stress models are considered highly necessary to represent work piece material plastic behavior under high-speed cutting conditions.

The Von-Mises yield criteria combined with the rate dependent Johnson-Cook (J-C) plastic method were employed to the finite element models.

Steps and Boundary Conditions

The work piece is constrained against any movement at the bottom edges (in 2D) or surface (in 3D), while tool is advancing with a constant velocity by penetrating into work piece. A constant step-time is applied to solve the problem for all simulations. Surface to surface contact algorithm was employed in the model by using penalty mechanical constraint.

For the heat transfer calculations, the following assumptions were made:

- The contact between the tool and the chip is thermally perfect. Hence a very large value of the interface heat transfer coefficient (h_{int}) was used and it was fixed to 1000 kW/m^{2}K (Filice *et al.,* 2007).
- The boundaries that are sufficiently away from the cutting zone remain at the room temperature (T_∞=20 ºC)
- The chip and the tool lose heat due to convection (h_∞=20 W/m^2 ºC) on the free surfaces of the work piece.
- Heat loss due to radiation is very small and it is neglected.

Thermal boundary conditions of the system can be defined as in Equations (1-4)

$$T = T_\infty \ (on\ ST) \tag{1}$$

$$-K\frac{\partial T}{\partial n} = h_{h\text{int}}(T - T_c)on\,Sc \tag{2}$$

$$-K\frac{\partial T}{\partial n} = h_\infty(T - T_\infty)on\,S_h \tag{3}$$

$$-K\frac{\partial T}{\partial n} = 0 \quad on\,S_o \tag{4}$$

Friction Model

A Coulomb's friction law was assumed to model the tool-chip and the tool-work piece contact zones. According to the Coulomb's law, the relative motion (slip) occurs at the contact point, when chip shear stress, τ is equal to or greater than the critical friction stress $\tau_t = \mu\sigma_n$. When τ is smaller than τ_t there is no relative motion and the contact point is in a state of stiff (Shet and Deng, 2000). The critical friction stress is determined by $\tau_t = \mu p\sigma_n$ where p is the normal pressure at the contact point where chip shear stress is determined, μ is the friction coefficient and σ_n is the threshold value related to material failure. If the σ_n is turning NST 37.2 steel with uncoated carbide cutting inserts. They employed artificial neural network- based tool wear predictive model and a genetic algorithm-based optimization model. Multi-layer, feed-force, back - propagation network was used in the predictive model, while the maximum metal removal rate (MRR) was used as the objective function. Samples of NST 37.2 steel bars with 25 mm diameter and 400 mm lengths work piece and Sandvice Coromant uncoated carbide inserts with International Standard Organization (ISO) designation SNMA 12406 were used.

Dry machining at different cutting conditions with cutting speed, feed rate and depth of cut ranging from 20.42-42.42 mm/min, 1.0-2.2 mm/rev and 0.2-0.8 mm, respectively were carried out. Eight passes of 50 mm length of cut were machined at each condition, the spindle power and tool wear (flank and nose) were measured during each cutting operation. They found out that the predictive model had acceptable accuracy and the optimum cutting parameters were obtained. Considerable work has been carried out on the NST 37-2 steel, but FEM investigation on the optimization of its machining parameters are nonexistent or scarce, hence this study.

Locally Rolled Steels in Nigeria

Steels are alloys that contain Iron as the major constituent and Carbon as the main alloying element in a small proportion. Other elements such as Nickel, Silicon, Chromium, Tin, Phosphorus, Molybdenum, and Manganese may also be present in small amounts. Steels are the most commonly used metallic materials for design and manufacture of structural components.

In Nigeria the steel industry comprises of Osogbo rolling mills, Ajaokuta rolling mills, Jos and Kastina rolling mills. These rolling mills make use of hot rolling process principle to produce majorly, thirteen grades of steel in form of wire, ribble and round rods with Nigerian Standards (NST) tagged as a designation system. Table 1 shows the grades of locally rolled steel produced in Nigeria with NST 37-2 being the most commercially produced (about 75%) due to its great mechanical properties, and it's wide use in mechanical and in building constructions

2.0 MATERIALS AND METHODS

For this study, finite element modelling of machining of NST 37-2 was developed using the commercial software ABAQUS. The following assumptions were made for the simulation, viz;

- The cutting velocity vector is normal to the cutting edge.
- The work piece material is a homogeneous polycrystalline, isotropic, and incompressible solid.
- The width of cut is much larger than the feed and both are constant.

Table 1: The Grades of Steel Production in Nigeria

Billet Qualities	C	Si	Mn	P	S	Cu	Cr	Ni	Sn
ST 33	0.100	0.160	0.600	0.040	0.040	0.250	0.100	0.010	0.050
ST 34-LC	0.120	0.280	0.500	0.450	0.050	0.700	0.700	0.010	
ST 37-2	**0.200**	**0.370**	**0.650**	**0.040**	**0.050**	**0.700**	**0.700**	**0.010**	**0.050**
ST 37-3	0.220	0.250	0.650	0.040	0.080	0.060	0.700	0.010	0.050
ST 44-2	0.220	0.350	0.650	0.045	0.055	0.300	0.300	0.010	
ST 44-3	0.200	0.350	0.680	0.045	0.050	0.250	0.300	0.010	0.010
ST 50-2	0.370	0.370	0.800	0.045	0.045	0.300	0.300	0.010	
ST 52-3	0.380	0.370	0.750	0045	0.050	0.300	0.300	0.010	
ST 60-2	0.490	0.370	1.800	0.045	0.045	0.300	0.300	0.010	
ST 60 MN	0.420	0.400	1.200	0.040	0.040	0.250	0.100	0.010	0.050
ST 65 MN	0420	0.400	1.600	0.040	0.040	0.250	0.100	0.010	
ST 70-2	0.490	0.400	1.200	0.045	0.045	0.300	0.100	0.010	
ST 70 MN	0.450	0.400	1.800	0.045	0.045	0.300	0.100	0.010	0.050

Source: (Osogbo Steel Rolling, 1982)

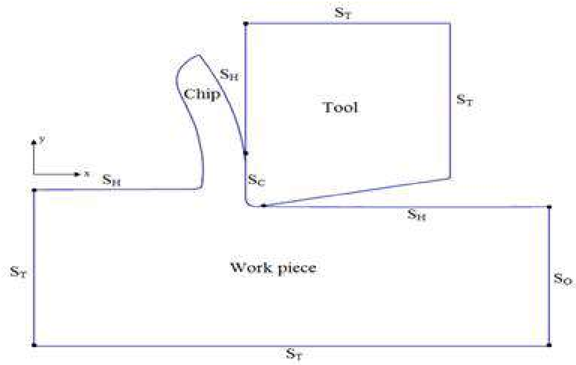

Figure 2: Thermal boundary conditions of the cutting system

set to a value rather than infinity, it is called a modified Coulomb frictional law. In this study, conventional Coulomb frictional law was used, by setting σ_n to infinity. Penalty method was used as a friction formulation. The tool is selected as a master object because it was defined as a form of rigid object. The work piece is defined as slave object.

Properties of Material and Simulation Approach

The physical properties of NST 37-2 steel are shown in Table 3. The material constants for Johnson-Cook material model and the damage model are presented in Table 4 and Table 5 respectively. These parameters were obtained experimentally.

The finite element simulation of cutting process was carried out by using ABAQUS/Explicit. The 2D and 3D Arbitrary Lagrangian and Eulerian formulation were applied. Metal cutting is affected by the type of work material, tool material and cutting conditions. The factors affecting chip formation are cutting speed, depth of cut, feed rate and tool rake angle. Adiabatic temperature boundary conditions were assumed. The effect of any cooling materials was not investigated in this study. The machining operation was assumed to be performed in air without application of any liquid coolant.

The combination of cutting conditions used in the simulation are listed below (for the cutting length of 10 mm and depth of cut of 0.5 – 1.0 mm):

- Cutting Speed range: 29.06 to 69.24 m/min at rake angle 15^0 and feed rate 1.0 mm/rev.
- More simulations were also run for 35.32, 59.34, 100.32, 169.56 m/min cutting speed and feed rate 1.65 mm/rev to compare the trend of rake face temperatures with experimental results reported by Sushil (2014).
- More simulations were run for 43.96 and 69.24 m/min cutting speed and with different feed rates (0.1, 0.2 and 0.8 mm/rev) to compare the trend of cutting force and thrust force with experimental results reported by Korka *et al.*, (2013).

3. RESULTS AND DISCUSSION

Figures 3 and 4 show the simulation results of the machining of NST 37-2 steel in 2 and 3 dimensions respectively.

In Figure 5, two zones in the chip formation area are visible, namely: the primary and the secondary zones. In the primary zone, there is appearance of separation of metal at a peak of the tool (orange). In this area, the tool exerts a compressive force on the material which is driven back and separates into two parts: the chip and the machined part. The temperature in the zone of separation tool part is around 183.5 °C, then it passes from 183.5 °C on the point of the tool to 244.6 °C and in all the progressions of the tool and finally the temperature releases itself through the chip.

Table 3: Mechanical Properties of NST 37-2 Steel

Properties	Values
Yield Strength (MN/m^2)	420.000
Elongation (%)	3.667
Reduction in Area (%)	15.050
Maximum Tensile Strength	683.449
Young Modulus (GPa)	28.565
Density (Kg/m^3)	8150

Table 4: Johnson-Cook Material Model Constants for NST 37-2 Steel

A (MPa)	B (MPa)	C	N	M	Room Temp	Melting Temp
340.00	683	0.062	0.11	0.42	20	1365

Table 5: Johnson-cook Damage Model Constants for NST 37.2 Steel

D1	D2	D3	D4	D5
0.026	0.853	-0.0388	0.3281	0.328

The heat generated within the interface is due to plastic work and friction in the work piece, and which is diffused in the tool. The chip formed during the simulations was observed to be continuous. It can be observed, that thermo-mechanical properties play an important role in shear localization and chip formation process in the machining of NST 37.2 steel.

Effects of cutting speed were observed for machining with 0.3 coefficient of friction, 15^0 rake angle at different feed rates. Table 6 shows the FEM results of cutting speed for the simulations.

Figures 6 – 9 show the effect of cutting speed on chip formation. Chips formed were observed to be continuous during machining at high cutting speeds. Rake face temperatures increase linearly with increasing cutting speed. Adiabatic shear bands were observed in the shear zone in machining at all higher speeds. Shear zone temperatures were observed to increase with increasing cutting speed. This is due to high speed of deformation with high cutting speed. Most of the heat resulting from the work done on the shear plane was carried away by the chips and a small amount of heat was conducted into the work piece.

Effect of cutting speed on cutting force

Cutting force in machining depends on the chip tool contact area and the shear strength of material in the primary deformation zone. Chip tool contact area reduces with increasing cutting speed contributing to a reduction in cutting force. Therefore cutting forces were observed to decrease with increasing cutting speeds. Work material in the primary shear zone is strengthened in the initial stages of chip formation as it undergoes strain hardening. Thermal softening was prominently observed as adiabatic shear band was formed. As temperatures in the shear zone increased with increasing cutting speeds, it led to higher rate of thermal softening in the shear zone. Work material was weakened rapidly at higher cutting speeds which was the reason for lower cutting forces. Power consumption was observed to increase linearly with increasing cutting speed.

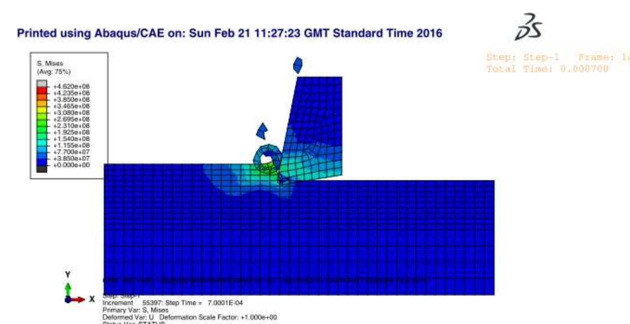

Figure 3: FEM simulation visualization results of machining NST 37.2 steel (2- dimensional view)

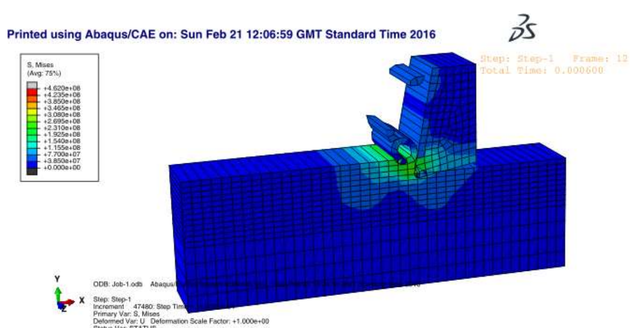

Figure 4: FEM simulation visualization results of machining NST 37.2 steel.(3-dimensional view)

Comparison of Cutting Tool temperature

The heat model was validated by comparing the predicted temperature distribution obtained in the simulation with the experimental results reported by Sushil (2014). In his experiment, temperature distributions were measured in depth of the cutting tool in a section corresponding with steady state conditions. Orthogonal dry cutting tests with air cooling were carried out on a lathe (HMT heavy duty lathe LTM-20) with the following parameters: cutting speed 35.32, 59.34, 100.32, 169.56 m/min, feed rate 1.65 mm/rev, cutting width 1.0 mm. The work piece material was EN8 steel and uncoated and coated carbide tools. This material has properties which are close to NST 37-2 steel properties with the same uncoated carbide tool properties. The temperature was observed to increase monotonically with increasing cutting speed. This is shown in Table 8

The FEM simulations with similar set of cutting conditions were run for comparisons.

Comparison of Cutting Force

Cutting force results obtained from FEM simulations were compared with experimental and analytical results reported by Zoltan *et al*, 2003. The cutting experiments were carried out on a precision turning lathe (type SN 560) and the work piece material was

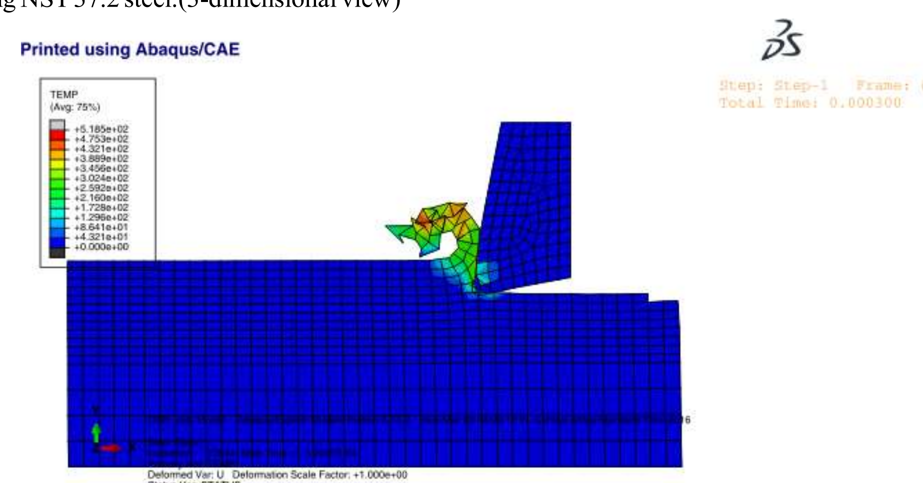

Figure 5: FEM of chip formation and deformation zones

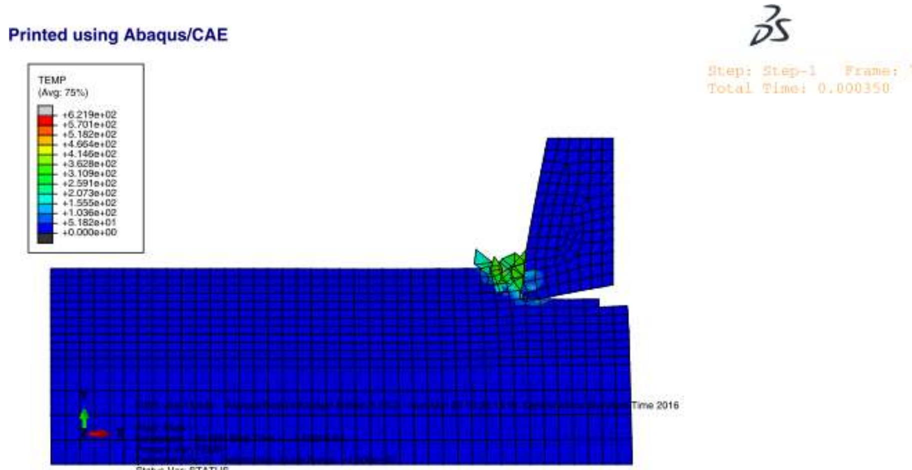

Figure. 6: Effect of cutting speed on chip formation and on shear zone temperatures in simulation of machining NST 37-2 steel with 0.3 (coefficient of friction), 15^0 rake angle at a feed rate of 1.0 mm/rev. Temperature variation with speed at 29.06 m/min.

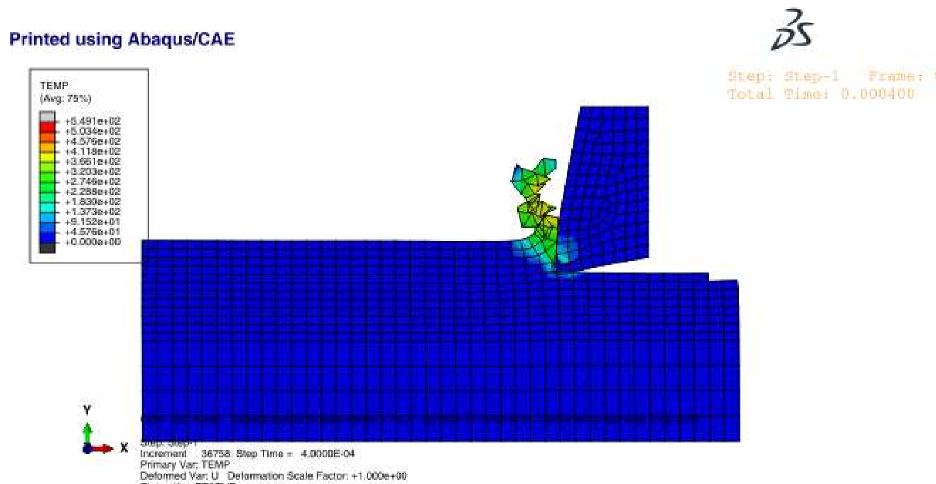

Figure 7: Effect of cutting speed on chip formation and on shear zone temperatures in simulation of machining NST 37-2 steel with 0.3 (coefficient of friction), 15^0 rake angle at a feed rate of 1.0 mm/rev. Temperature variation with speed at 35.76 m/min.

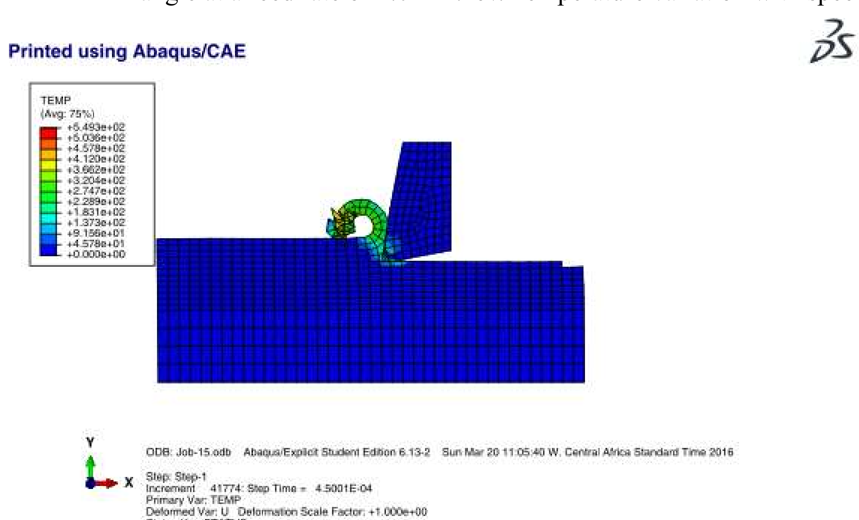

Figure 8: Effect of cutting speed on chip formation and on shear zone temperatures in simulation of machining NST 37-2 steel with 0.3 (coefficient of friction), 15^0 rake angle at a feed rate of 1.0 mm/rev. Temperature variation with speed at 42.46 m/min.

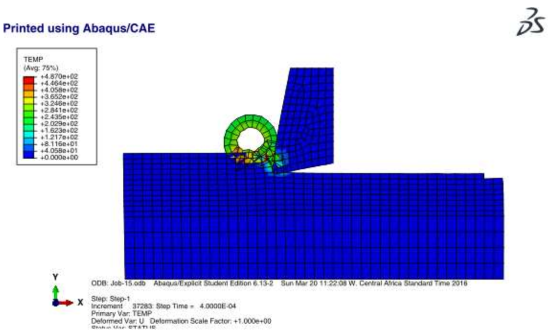

Figure 9: Effect of cutting speed on chip formation and on shear zone temperatures in simulation of machining NST 37-2 steel with 0.3µ, 15^0 rake angle at a feed rate of 1.0 mm/rev. Temperature variation with speed at 69.24 m/min.

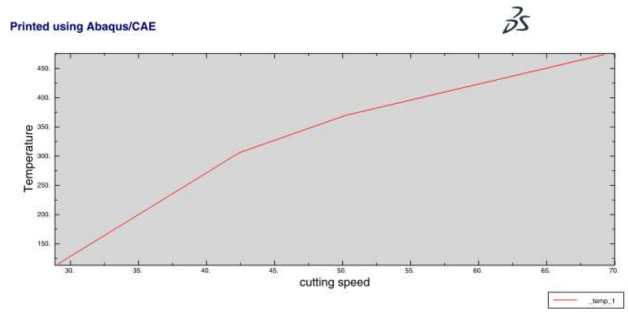

Figure 10: The plot of the effect of cutting speed on shear zone temperatures in simulation of machining NST 37-2 steel with 0.3µ 15^0 rake angle at a feed rate of 1.0 mm/rev.

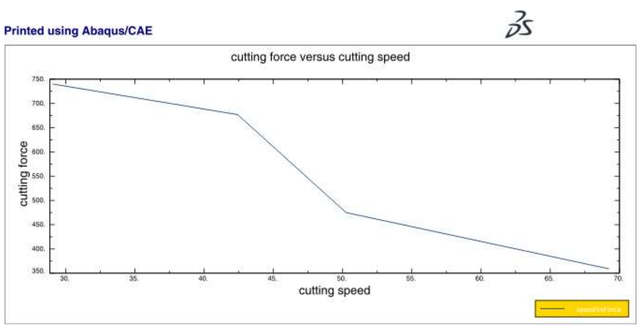

Figure 11: Effect of cutting speed on cutting force in simulation of machining NST 37-2 steel with 0.3µ, 15^0 rake angle at a feed rate of 1.0 mm/rev

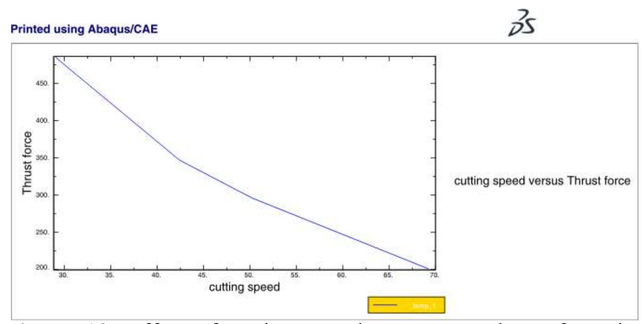

Figure 12: Effect of cutting speed on average thrust force in simulation of machining NST 37-2 steel with 0.3µ, 15^0 rake angle at a feed rate of 1.0 mrev

Table 7: FEM Result for the Different Cutting Speeds.

No	Cutting speed (m/min)	Feed rate (mm/rev)	Depth of cut (mm)	Temperature (^{0} C)	Cutting Force (N)	Thrust Force (N)
1	29.06	1.5	0.5	318	436	387
		2.0		341	446	409
2	29.06	1.0	0.8	320	497	423
		1.5		345	502	436
		2.0		362	515	445
		2.5		370	521	449
3	35.76	1.0	0.8	348	483	406
		1.5		372	489	418
		2.0		386	480	427
		2.5		392	472	432
4	42.46	1.0	0.8	380	474	387
		1.5		389	480	391
		2.0		400	492	401
		2.5		406	501	407
5	49.16	1.0	0.8	394	463	371
		1.5		406	471	380
		2.0		414	477	385
		2.5		421	486	389
6	55.86	1.0	0.8	403	445	356
		1.5		409	456	362
		2.0		417	468	368
		2.5		424	472	376
7	69.24	1.5	1.0	430	459	366
		2.0		423	470	377

mild steel. Machining was carried out using standard high speed steel tools with a 25 mm square shank. Cutting process was carried out without the use of cutting fluid, and this was continued until the flank wear of the tool achieved a maximum value VBmax= 0,6 mm.

Based on the experimental data, cutting conditions: cutting depth, t = 0.5mm, feed rate, f = 0.2mm/rev, cutting speeds 43.96 and 69.24 and 87.92 m/min were also selected for simulations. The results are presented in Table 9

4. CONCLUSION

In this study, finite element simulation approach has been used to study the effect of machining conditions on chip formation, temperatures, equivalent plastic strain, residual stress distribution, cutting and thrust forces in machining of Nigeria locally rolled NST 37-2 steel by using a commercial software ABAQUS.

The dynamics, explicit Arbitrary Lagrangian Eulerian method was utilized to develop the FEM simulation model with a round edge uncoated carbide cutting tool. The Johnson-Cook work material model and a detailed friction model were also used, and work material flow around the round edge of the cutting tool was simulated in conjunction with an adaptive meshing scheme. The findings of this investigation are as follows:

- The chip formed throughout the simulation was mainly continuous,
- The cutting force and the thrust force were observed to decrease with increasing cutting speed. The temperature was found to increase with increasing cutting speed.
- Close comparisons of the FEM results with experimental data, and observations of chip formation procedure validate assumptions and application of Johnson cook material models and columbic friction model.

Table 8: Temperatures comparison of FEM simulation of NST 37-2 steel simulations with experiment at 1.65 mm/rev and 1.0 mm width of cut.

Cutting speed (m/min)	Temperature (^{0}C) Experiment	Temperature (^{0}C) FEM
35.32	380	376
59.34	410	415
100.32	500	498
169.56	605	609

Table 9: Cutting forces observed at a range of cutting speeds 43.69 to 109.9 m/min, feed rate of 0.2 mm/rev and 0.5 depth of cut in FEM simulations and experimental studies.

Cutting speed (m/min)	Experiment Cutting Force (N)	Experiment Thrust Force (N)	FEM Cutting Force (N)	FEM Thrust Force (N)
43.96	460	355	468	332
69.24	390	285	393	291
87.92	385	280	390	286
109.9	370	266	374	267

5. REFERENCES

Ali Moaz H., Khidir B. A., Ansari, M. N. M., Mohamed B. (2013). FEM to predict the effect of feed rate on surface roughness for face milling of titanium alloy, Journal of Housing and Building National Research Center, 7: 263-269.

Cenk K. (2009). Modelling and simulation of metal cutting by finite element Method, A Thesis Submitted to the Graduate School of Engineering and Sciences of İzmir Institute of Technology.

Dhananjay J. (2004). Finite element simulation of machining a nickel-based superalloy - inconel 718, A MSc. Thesis Submitted to Graduate College of the Oklahoma State University, 45-48.

Everton Ruggeri S. A. (2014). Finite Element Simulation of Chip Formation in Machining Process, State Universityof Santa Catarina UDESL CCT, ISSN 8219-710, 4-5.

Fadare D. A. and Asafa T. B. (2010). Optimization of Turning NST 37.2 Steel with Uncoated Carbide Cutting Tools, Journal of Nigerian Institution of Mechanical Engineers, 2, 1: 31-40.

Filice L., Micari F., Rizutti S. and Umbrello D. (2007). A Critical Analysis on the Friction Modelling In Orthogonal Machining, International Journal of Machine Tools and Manufacturing, 47: 709-714.

Malomo B. O., Ibitoye S. A., Adekoya L. O. (2010). The Study of Elastic- Plastic Fatigue Behaviour of Nigerian Rolled NST 37-2 Steel, International Journal of Engineering Research in Africa, ISSN 1663-4144, 3: 28-41.

Korka Z.I., C-O, Miclosina and V. Cojocaru (2013). An Experimental Study of the Cutting Forces in Metal Turning, Journal of Universita II Eftimie Murgu Resita Anul XX, 2, ISSN 1453-7397.

Kovac P., B. Savkovic, B. Serdar and M. Sekalic (2011). Modelling Mechanical and Thermal Load of Cutting Tool. ACTA T.C. - Bulletin of Engineering, 78-80.

Shet C. and Deng X. (2000). Finite Element Analysis of the Orthogonal Metal Cutting Process, Journal of Materials Processing Technology, 105: 95-109.

Sushil D. G. (2014). Temperature Measurement of a Cutting Tool in Turning Process by Using Tool Work Thermocouple, International Journal of Research in Engineering and Technology ISSN: 2319-1163 ISSN: 2321-7308.

Tounsi N., Vincenti J., Otho A., Elbestawi M.A. (2002). From the Basics of Orthogonal Metal Cutting Toward the Identification of the Constitutive Equation, International. Journal of Machine Tools and Manufacture, 42: 1373-1383.

Zhang Y. C., T. Mabrouki, D. Nehas and Y.D. Gong (2011). Chip formation in orthogonal cutting considering interface limiting shear stress and damage evolution based on fracture energy approach, Journal of Finite Elements Analysis and Design,